SLATT UNDERGRADUATE RESEARCH FELLOWSHIP
FINAL REPORT

SCHOLAR NAME:	Evan Maxwell Bursch
FACULTY ADVISOR:	Ryan McClarren
PROJECT PERIOD:	2022
PROJECT TITLE:	High-Order Implicit Shock Tracking For Inertial Confinement Nuclear Fusion
CONNECTION TO ONE OR MORE ENERGY-RELATED RESEARCH AREAS (CHECK ALL THAT APPLY):	**( ) Energy Conversion and Efficiency** **(X) Sustainable and Secure Nuclear** **( ) Smart Storage and Distribution** **( ) Transformation Solar** **( ) Sustainable Bio/Fossil Fuels** **( ) Transformative Wind**

MAJOR GOALS AND ACCOMPLISHMENTS

Summarize your research goals and provide a brief statement of your accomplishments (no more than 1-2 sentences). Indicate whether you were able to accomplish your goals by estimating the percentage completed for each one. Use the next page for your written report.

RESEARCH GOALS	ACTUAL PERFORMANCE AND ACCOMPLISHMENTS	% OF GOAL COMPLETED
Asses compatibility of radiation hydrodynamics (RHD) with the computational method framework of High Order Implicit Shock Tracking (HOIST)	Deriving equations for and modeling RHD systems using the HOIST framework was accomplished. Using models based on published, standardized, RHD equations, the RHD physics was able to be incorporated into the HOIST system.	100 %
Model discontinuities in RHD problems with HOIST	The main draw of HOIST is its ability to handle discontinuities in finite element methods, which were successfully applied to RHD problems.	100 %
Demonstrate use and further promise of RHD HOIST.	The research showed viability in lower-dimensional RHD problems which promise continued benefits in higher-dimensional simulation problems.	100 %

RESEARCH OUTPUT

Please provide any output that may have resulted from your research project. You may leave any and all categories blank or check with your faculty advisor if you are unsure how to respond.

CATEGORY	INFORMATION
EXTERNAL PROPOSALS SUBMITTED	(Sponsor, Project Title, PIs, Submission Date, Proposal Amount)
EXTERNAL AWARDS RECEIVED	(Sponsor, Project Title, PIs, Award Date, Award Amount)
JOURNAL ARTICLES IN PROCESS OR PUBLISHED	(Journal Name, Title, Authors, Submission Date, Publication Date, Volume #, Page #s)
BOOKS AND CHAPTERS RELATED TO YOUR RESEARCH	(Book Title, Chapter Title, Authors, Submission Date, Publication Date, Volume #, Page #s)
PUBLIC PRESENTATIONS YOU MADE ABOUT YOUR RESEARCH	(Event, Presentation Title, Presentation Date, Location) 10th International Conference on Numerical Methods for Multi-Material Fluid Flow, “Radiation Hydrodynamics Modeling With High-Order Implicit Shock Tracking”, August 23rd, 2022, Zurich, Switzerland
AWARDS OR RECOGNITIONS YOU RECEIVED FOR YOUR RESEARCH PROJECT	(Purpose, Title, Date Received)
INTERNAL COLLABORATIONS FOSTERED	(Name, Organization, Purpose of Affiliation, and Frequency of Interactions) Professor Matthew Zahr, Notre Dame, He and Professor McClarren co-advised me on this project and we interacted multiple times a month.
EXTERNAL COLLABORATIONS FOSTERED	(Name, Organization, Purpose of Affiliation, and Frequency of Interactions)
WEBSITE(S) FEATURING RESEARCH PROJECT	(URL) https://www.math.uzh.ch/multimat2021/index.php?id=99

OTHER PRODUCTS AND SERVICES (e.g., media reports, databases, software, models, curricula, instruments, education programs, outreach for ND Energy and other groups)	(Please describe each item in detail)

RESEARCH EXPERIENCE

Please let us know what you thought of your research experience: Did this experience meet your expectations? Were lab personnel helpful and responsive to your needs? What else could have been done to improve your experience or achieve additional results?

The Slatt Fellowship allowed me the opportunity to dedicate my time and energy towards research instead of an on campus job. Also, I was able to present my research at an international conference, the 10th International Conference on Numerical Methods for Multi-Material Fluid Flow, in Zurich, Switzerland. I was the only undergraduate student who gave an oral presentation, along with a couple graduate students and numerous professors and researchers from universities, laboratories, and companies around the world. It was an amazing experience and helped broaden my understanding of the international research community and built up skills in scientific writing and presenation. Thank you very much to the Slatt family and to ND Energy for this program and the blessings it has brought me in my research and personal life.

FINAL WRITTEN REPORT

(Please use the space below to describe your research project and objectives, any findings and results you can share, and graphs, charts, and other visuals to help us understand what you achieved as a result of this research experience.)

Traditional methods for modeling radiation hydrodynamics can present problems when encountering discontinuities. By leveraging shock tracking, an alternative method to shock capturing, we aim to generate a mesh such that element faces align with shock surfaces and other non-smooth features to perfectly represent them with the inter-element jumps in the solution basis, e.g., in the context of a finite volume or discontinuous Galerkin (DG) discretization. These methods lead to high-order approximations of high-speed flows and do not require nonlinear stabilization or extensive refinement in non-smooth regions because, once the non-smooth features are tracked by the mesh, the high-order solution basis approximates the remaining smooth features.

In my research, we have demonstrated the viability of radiation hydrodynamics modeling using the High Order Implicit Shock Tracking (HOIST) method that re-casts the geometrically complex problem of generating a mesh that conforms to all discontinuity surfaces as a PDE-constrained optimization problem. The optimization problem seeks to determine the flow solution and nodal coordinates of the mesh that simultaneously minimize an error-based indicator function and satisfy the discrete flow equations. A DG discretization of the governing equations is used as the PDE constraint to equip the discretization with desirable properties: conservation, stability, and high-order accuracy. By using high-order elements, curved meshes are obtained that track curved shock surfaces to high-order accuracy. The optimization problem is solved using a sequential quadratic programming method that simultaneously converges the mesh and DG solution, which is critical to avoid nonlinear stability issues that would come from computing a DG solution on an unconverged (non-aligned) mesh. The method is used to solve several well-known problems in radiation hydrodynamics including subsonic Marshak waves and radiation shock solutions.

In the future, this reseach can be expanded to higher dimensions for more interesting applications and more benefit to the inertial confinement fusion research community. This modeling has shown promise and an opportunity for further developments to address issues in high-fidelity modeling and simulation for fusion experiments in the pursuit of nuclear fusion energy research.